Bell's Theorem, Amplitudes, and Trigonometry

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Abstract

Bell's theorem adds probabilities when it should use amplitudes. With amplitudes and trigonometric identities, a theory in which the result at one detector does not depend on the other detector's setting or outcome can produce the correct statistics for a Bell test. When a photon's polarization before a measurement is a hidden variable, Bell's theorem thus does not rule out local realism.

1 Introduction

In general, it is necessary to do arithmetic for quantum events by adding and multiplying complex amplitudes, rather than the probabilities that result after applying the Born rule. Failing to do so can result in missing interference terms, as in the double-slit experiment, since the operations of addition and taking the complex conjugate do not commute with each other over the complex numbers[1].

The original proof of Bell's theorem[2] involves adding quantum probabilities: expectation values come from the sum of probabilities, so either adding or multiplying those involves adding probabilities.

2 Bell's Theorem

For a Bell test using photon polarization and the Bell state $|\Phi^+\rangle$, an iteration of the experiment will involve two photons polarized at some angle X, and detectors d_A set at an angle θ_A and d_B at angle θ_B. The relative angle between X and θ_A is A, and between X and θ_B is B.

To obey the classical version of Malus's law[3], the probability that the photon at d_A passes through is $cos^2(A)$, the probability it is blocked is $sin^2(A)$, and the same is true for d_B and B. The probability that the results at d_A and d_B agree is $cos^2(A-B)$ and that they disagree is $sin^2(A-B)$[4].

Bell's theorem claims that a local realist theory, in which the result at d_A depends on A but not B, and at d_B depends on B but not A, cannot reproduce those statistics.

Using trigonometric identities, and doing arithmetic on amplitudes rather than probabilities, it is possible to design such a theory.

3 A Local Realist Solution

We will use $cos(x)$ as the amplitude that a photon polarized with relative angle x passes through a linear polarizer, and $isin(x)$ as the amplitude that it does not. These satisfy the desired probabilities for individual measurements, after applying the Born rule.

It may be possible to use other functions, or that there is some obstruction to using these, but it should demonstrate that local realism may be a lot more plausible than Bell's theorem seems to indicate.

To get the $cos^2(A-B)$ and $sin^2(A-B)$ results, we have to use the convention that exactly one of A and B is negative. Maybe there is a good reason for that, but it makes the math work.

The following trigonometric identities[5] will be useful:

$$cos(-x) = cos(x),$$
$$sin(-x) = -sin(x),$$
$$cos(A)cos(B) + sin(A)sin(B) = cos(A-B),$$
$$sin(A)cos(B) - cos(A)sin(B) = sin(A-B).$$

The amplitude of passing through at both d_A and d_B is $cos(A)cos(-B) = cos(A)cos(B)$, and of being blocked at both is d_A and d_B is $isin(A)isin(-B) = sin(A)sin(B)$. If we multiply each of those by their complex conjugates and then add them together, we will be missing a term. So, it appears important to add the amplitudes before computing a probability, in which case we get the right result.

The amplitude of passing through d_A but not d_B is $cos(A)isin(-B) = -icos(A)sin(B)$, and d_B but not d_A is $isin(A)cos(-B) = isin(A)cos(B)$. Adding them together results in a subtraction, since they have opposite signs, so we again get the right probability after applying the Born rule.

4 Conclusion

These results suggest that Bell's theorem does not disprove local realism.

Here the polarization of the photons before measurement, X, is a hidden variable. It leads to a quantized measurement outcome without itself being quantized, which is relevant for other arguments against hidden variables, such as the Kochen-Specker theorem[6].

References

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