Separable Bell Pairs via Interference

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Abstract

The original proof of Bell's theorem assumes a lack of interference when calculating probabilities for hidden-variable theories. Accounting for interference, by doing arithmetic on probability amplitudes and only then applying the Born rule, allows a local-realist theory to compute the correct expectation values for Stern-Gerlach and photon polarization Bell tests. While standard quantum mechanics equates entanglement with inseparability for pure states, the relevant Bell pairs are separable. Bell tests then do not provide evidence against local realism, but only against local-realist theories that ignore interference.

1 Introduction

Bell's theorem[1] attempts to show that local-realist theories cannot reproduce specific predictions of quantum mechanics. Its original proof assumes that local-realist theories do not use interference, via the idea of factorizability[2], as shown in Section 2.

Section 3.1 shows that local-realist theories[3] can correctly predict the expectation values of Stern-Gerlach Bell tests with spin-$\frac{1}{2}$ particles. An extension of similar work from previous papers[4][5], for Bell tests with photon polarization, is in Section 3.2.

Section 3.3 reviews a key aspect of the examples in Sections 3.1 and 3.2, interference, which makes it necessary to do arithmetic on probability amplitudes, before applying the Born rule.

Standard quantum mechanics agrees with Section 3 for pairs of events, like $P(|1,1\rangle \text{ or } |-1,-1\rangle)$, but disagrees for individual events like $P(|1,1\rangle)$, as shown in Section 4.1. Many papers on Bell tests report only the results for pairs of outcomes, and not individual outcomes.

Section 4.2 argues that standard quantum mechanics cannot always predict the correct statistics, using only the quantum state. A proof that hidden variables are required to have a separable Bell state, given the failure of Bell's theorem in Section 2, and the states in Sections 3.1 and 3.2, is in Section 4.3.

The conclusion in Section 5 argues that Bell tests do not provide evidence against local realism, but only against local-realist theories that ignore interference.

2 Factorizability

The notion of factorizability in Bell's theorem[2] assumes that a local-realist theory does not use interference. The basic idea is that, where E is the expectation value,

$$E(A,B) = E(A)E(B). \tag{1}$$

If there is no interference, that works:

$$\begin{gathered}
E(A,B) = P(|{\downarrow\downarrow}\rangle) + P(|{\uparrow\uparrow}\rangle) - P(|{\downarrow\uparrow}\rangle) - P(|{\uparrow\downarrow}\rangle) \\
E(A) = P(|{\downarrow_A}\rangle) - P(|{\uparrow_A}\rangle) \\
E(B) = P(|{\downarrow_B}\rangle) - P(|{\uparrow_B}\rangle) \\
E(A)E(B) = \\
P(|{\downarrow_A}\rangle)P(|{\downarrow_B}\rangle) + P(|{\uparrow_A}\rangle)P(|{\uparrow_B}\rangle) - P(|{\downarrow_A}\rangle)P(|{\uparrow_B}\rangle) - P(|{\uparrow_A}\rangle)P(|{\downarrow_B}\rangle).
\end{gathered} \tag{2}$$

However, if there is interference, then that equality can fail: $P(|{\downarrow\downarrow}\rangle \text{ or } |{\uparrow\uparrow}\rangle)$ and $P(|{\downarrow\uparrow}\rangle \text{ or } |{\uparrow\downarrow}\rangle)$ will have extra terms compared to the versions without interference. For the first case, that is

$$\begin{gathered}
P(|{\downarrow\downarrow}\rangle \text{ or } |{\uparrow\uparrow}\rangle) = |(a+bi)|{\downarrow\downarrow}\rangle + (c+di)|{\uparrow\uparrow}\rangle|^2 \\
(a+c)^2 + (b+d)^2 \\
a^2 + b^2 + c^2 + d^2 + 2ac + 2bd
\end{gathered} \tag{3}$$

with interference, and

$$\begin{gathered}
P(|{\downarrow\downarrow}\rangle \text{ or } |{\uparrow\uparrow}\rangle) = P(|{\downarrow\downarrow}\rangle) + P(|{\uparrow\uparrow}\rangle) \\
|(a+bi)|{\downarrow\downarrow}\rangle|^2 + |(c+di)|{\uparrow\uparrow}\rangle|^2 \\
a^2 + b^2 + c^2 + d^2
\end{gathered} \tag{4}$$

without.

In Sections 3.1 and 3.2, the extra terms do not cancel out when computing the expectation values. So, if there is interference in a Bell test, the standard proof of Bell's theorem fails.

3 Bell Tests With Hidden Variables

3.1 Stern-Gerlach

It is possible to reproduce the expectation values of a Stern-Gerlach Bell test using local realism, one outcome per measurement, and no superdeterminism or retrocausality.

The probability amplitudes for measuring devices A and B are

$$\begin{gathered}
\psi_A = \cos\left(\frac{A}{2}\right)\left|\uparrow\right\rangle + i\sin\left(\frac{A}{2}\right)\left|\downarrow\right\rangle \\
\psi_B = -i\sin\left(\frac{B}{2}\right)\left|\uparrow\right\rangle + \cos\left(\frac{B}{2}\right)\left|\downarrow\right\rangle.
\end{gathered} \tag{5}$$

[6] has those, with a sign flip on the angles for ψ_B, as rows in a table for a rotation about the x-axis in a Stern-Gerlach experiment with spin-$\frac{1}{2}$ particles. [7] contains some related ideas.

It is important to account for interference, by doing arithmetic on probability amplitudes before applying the Born rule, as in [4] and [5]. See Section 3.3.

[5] describes how the way a Bell pair is created can explain why the sign on one angle is flipped, for photons. Maybe something similar would work here.

Dividing the angle by two makes sense, since an angle of 2π flips a sign, while 4π restores the original state, as is characteristic of spin-$\frac{1}{2}$ particles[6].

Some useful trigonometric identities[8] are:

$$-\cos(A-B) = \sin^2\left(\frac{A-B}{2}\right) - \cos^2\left(\frac{A-B}{2}\right) \tag{6}$$

$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B) \tag{7}$$

$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B) \tag{8}$$

The probability amplitude for agreeing, based on (7), is

$$\begin{gathered} |{\downarrow\downarrow}\rangle + |{\uparrow\uparrow}\rangle \\ i\sin\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right) + \cos\left(\frac{A}{2}\right)\left(-i\sin\left(\frac{B}{2}\right)\right) \\ i\sin\left(\frac{A-B}{2}\right) \end{gathered} \tag{9}$$

The probability amplitude for disagreeing, based on (8), is

$$\begin{gathered} |{\uparrow\downarrow}\rangle + |{\downarrow\uparrow}\rangle \\ \cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right) + i\sin\left(\frac{A}{2}\right)\left(-i\sin\left(\frac{B}{2}\right)\right) \\ \cos\left(\frac{A-B}{2}\right) \end{gathered} \tag{10}$$

Applying the Born rule to those probability amplitudes leads to the right expectation value, $-\cos(A-B)$[2].

To incorporate hidden variables as in [5], it may be possible to view each particle's magnetic moment as a hidden variable. The angles in the probability amplitudes are then between those and each magnet's magnetic field. [6] indirectly mentions those values when describing the probability amplitudes for a Stern-Gerlach experiment on spin-$\frac{1}{2}$ particles.

3.2 Photon Polarization

[4] and [5] discuss a Bell test with photons, with calculations similar to Section 3.1: there is interference, one angle is flipped in the pair, and the probability

amplitudes are

$$\begin{aligned} \psi_A &= \cos(A-y)|1\rangle + i\sin(A-y)|-1\rangle \\ \psi_{B-y} &= \cos(B-y)|1\rangle - i\sin(B-y))|-1\rangle. \end{aligned} \tag{11}$$

Here A and B are the detector settings, and y, a hidden variable, is the linear polarization of the photon sent to that detector[5].

The probabilities for agreeing and disagreeing match the standard quantum mechanical predictions, $\cos^2(A-B)$ and $\sin^2(A-B)$. Those previous papers did not compute other probabilities, like $P(|1,1\rangle \text{ or } |1,-1\rangle)$.

For a combination of two measurement outcomes, $|1,1\rangle$ and $|-1,-1\rangle$ interfere with each other, and $|1,-1\rangle$ and $|-1,1\rangle$ do. Other pairs produce the correct results with or without interference, due to the way that the real and complex parts are squared separately in $|z|^2$. For example,

$$\begin{gathered} P(|1,1\rangle \text{ or } |1,-1\rangle) \\ |\cos(A-y)\cos(B-y) - i\cos(A-y)\sin(B-y)|^2 \\ \cos^2(A-y)\cos^2(B-y) + \cos^2(A-y)\sin^2(B-y) \\ \cos^2(A-y)\big(\cos^2(B-y) + \sin^2(B-y)\big) \\ \cos^2(A-y) \end{gathered} \tag{12}$$

That makes sense, because it is equal to the probability that the photon at A is measured as $|1\rangle$.

Probabilities of individual events, such as $P(|1,1\rangle) = \cos^2(A-y)\cos^2(B-y)$, correspond to treating the measurements at each detector as independent events. Entanglement means that y occurs in the expressions for both detectors, so they are related in that way.

The probabilities for three possible outcomes require that there is no interference for those events:

$$P(|1,1\rangle \text{ or } |-1,1\rangle \text{ or } |-1,-1\rangle) = 1 - P(|1,-1\rangle). \tag{13}$$

Otherwise, at $A-y = B-y = \frac{\pi}{4}$, that probability is greater than 1.

The total probability equals 1, following [4] and [5]: the probability amplitude for agreeing is real, and the one for disagreeing is imaginary, so both are separately squared, then added, to get the correct result. That probability is also correct without interference.

Section 4.1 shows that the hidden-variable probabilities for single events, like $|1,1\rangle$, do not match the standard predictions of quantum mechanics.

One nice property of this hidden-variable approach is that it provides a direct explanation of Malus's law[9]: the intensity of linearly-polarized light passing through a polarizer follows $\cos^2(A-y)$.

3.3 Interference

[5] speculates that interference occurs within subsystems and when it is possible to predict the effects of waves. [10] says that quantum computers do

arithmetic on probability amplitudes because they are isolated systems and the transformations are unitary. [11] suggests the heuristic of when an event can occur in several different ways, using the example of the double-slit experiment. It would be helpful to have one explanation that works in general.

It may be possible for an experiment to test whether there should be interference in the probabilities of agreeing and disagreeing in a photon polarization Bell test, as in Section 3.2.

Spontaneous parametric down-conversion can be used to send out two entangled photons in Bell tests[12]. It seems that a modified version of such a setup might instead have two sources, facing in opposite directions, sending out pairs of photons that are not entangled but have linear polarizations at the same angle, possibly with a sign flipped.

Then, if the latter experiment does not have interference, Section 2 indicates that the expectation values should differ by a term of

$$4\cos(A)\cos(B)\sin(A)\sin(B). \tag{14}$$

At $A = B = \frac{\pi}{4}$, a normal Bell test would have an expectation value of 1, and the version without interference should have an expectation value of 0.

The original Bell test setup is, under idealized conditions and in a deterministic interpretation, like sending a single photon to the same polarizer, twice. Without decoherence or experimental imprecision, $A = B$ would guarantee agreement. The modified setup is like running two separate trials at the same angle, when, across all possible angles, there is no bias toward agreeing or disagreeing.

4 Bell Tests Without Hidden Variables

4.1 Disagreement With Hidden Variables

It is possible to convert the hidden-variable states from Section 3.2 to probabilities of states without hidden variables, by applying integrals. For example, $P(|1,1\rangle) = \cos^2\big(A-y\big)\cos^2\big(B-y\big)$, and

$$\begin{gathered}\frac{1}{2\pi}\int_0^{2\pi}\cos^2\big(A-y\big)\cos^2\big(B-y\big)\,dy = \\ \frac{1}{4}+\frac{1}{8}\cos(2A-2B)\end{gathered} \tag{15}$$

via SageMath 9.5[13]. The resulting probabilities are

$$\begin{aligned}P(|1,1\rangle) = P(|-1,-1\rangle) &= \frac{1}{4}+\frac{1}{8}\cos(2A-2B)\\ P(|1,-1\rangle) = P(|-1,1\rangle) &= \frac{1}{4}-\frac{1}{8}\cos(2A-2B)\end{aligned} \tag{16}$$

Those differ from the predictions of standard quantum mechanics. For example, at $A = B = 0$, the first pair are each $\frac{3}{8}$ and the second are $\frac{1}{8}$, instead of the standard $\frac{1}{2}$ and 0.

Following Section 3.3, interference for $P(|1,1\rangle \text{ or } |-1,-1\rangle)$ and $P(|1,-1\rangle \text{ or } |-1,1\rangle)$ requires that events like $|1,1\rangle$ are different than in standard quantum mechanics, as $P(|1,1\rangle \text{ or } |-1,-1\rangle) \neq 2P(|1,1\rangle)$. Section 3.2 shows that those probabilities are reasonable in a hidden-variable model.

It may make sense that sometimes the results disagree even when $A = B$. If there is decoherence, a pair of photons that each had an initial polarization of y may have angles of $y+m$ and $y+n$ when they reach the polarizers. If $m \neq n$, it seems possible that the measurements will disagree. If $A - y$ is close to 45°, maybe that would be even more likely. See [3] for related ideas.

4.2 Measurement Statistics

It should be possible for experiments to test whether the hidden-variable predictions or standard predictions are correct. But, many papers on Bell test experiments seem to only report the expectation values, or the percentage of outcomes that agree versus disagree, rather than individual events like $|1,1\rangle$. Again, two-event outcomes like $P(|1,1\rangle \text{ or } |-1,-1\rangle)$ will be the same in both approaches.

Even if the probabilities are correct in a non-hidden-variable model, the quantum state does not provide enough information to know which probability to assign $|1,-1\rangle$ versus $|-1,1\rangle$, in a state $s|1,1\rangle + t|-1,-1\rangle$, with $|s|^2 \neq |t|^2$.

In standard quantum mechanics[14], those probabilities will be $|s|^2 \sin^2\big(A-B\big)$ and $|t|^2 \sin^2\big(A-B\big)$. But, it is not clear which should belong to $|1,-1\rangle$ and which to $|-1,1\rangle$. The prediction is that one detector will have results biased toward $|1\rangle$, and the other toward $|-1\rangle$, but the state does not indicate which way.

For a different example of when the quantum state does not provide enough information to compute probabilities, based on the states from Section 3.2, see [15].

4.3 Impossibility Proof

Whether or not there is interference in $P(|1,1\rangle \text{ or } |-1,-1\rangle)$ or $P(|1,-1\rangle \text{ or } |-1,1\rangle)$, it is impossible to have a separable Bell state without hidden variables.

$P(|1\rangle) = P(|-1\rangle) = \frac{1}{2}$, by doing integrals on the hidden-variable probabilities, $\cos^2\big(A-y\big)$ and $\sin^2\big(A-y\big)$. That distribution matches unpolarized light, and it agrees with $\frac{1}{\sqrt{2}}\Big(|1,1\rangle + |-1,-1\rangle\Big)$, where each qubit has a $\frac{1}{2}$ probability of ending up as $|1\rangle$ or as $|-1\rangle$.

To get those single-qubit probabilities, there are three possibilities for each of the probability amplitudes for $|1_\mathrm{A}\rangle$, $|-1_\mathrm{A}\rangle$, $|1_\mathrm{B}\rangle$, and $|-1_\mathrm{B}\rangle$:

1) Both the real and imaginary parts are constant.

2) The real part is constant, and the imaginary part is a function of the angle, or vice versa.

3) The probability amplitude is $\frac{1}{\sqrt{2}}e^{ix}$, where x is a function of that angle.

Examples of 1) would be $\frac{1}{2}+\frac{1}{2}i$, or $\frac{1}{\sqrt{2}}$. But, for any of the single-qubit choices like $|1_\mathrm{A}\rangle$, at least one two-qubit state, such as $|1_\mathrm{A},1_\mathrm{B}\rangle$, will either be a constant, or just a function of the other angle, rather than both A and B. Then, it will not match the expected probabilities, which are all functions of both A and B.

For 2), it is not possible to get $P(|1\rangle)=\frac{1}{2}$. Applying the Born rule would result in

$$\begin{aligned} f(\theta)^2+c^2 &= \frac{1}{2} \\ f(\theta)^2 &= \frac{1}{2}-c^2 \\ f(\theta)^2 &= d \\ f(\theta) &= \sqrt{d} \end{aligned} \tag{17}$$

for some function $f(\theta)$ of the relevant angle A or B, and constants c and d. But, that means $f(\theta)$ would be constant, which becomes 1).

The remaining case is when all probability amplitudes match 3). Then $|1,1\rangle$ has a probability amplitude of

$$\begin{gathered} \frac{1}{\sqrt{2}}e^{is}\cdot\frac{1}{\sqrt{2}}e^{it} \\ \frac{1}{2}e^{i(s+t)} \end{gathered} \tag{18}$$

where s is a function of A, and t is a function of B. Setting $u=s+t$ and applying the Born rule gives

$$\frac{1}{4}\cos^2(u)+\frac{1}{4}\sin^2(u)=\frac{1}{4} \tag{19}$$

when the probability is again a constant, rather than a function of both A and B.

5 Conclusion

The original proof of Bell's theorem assumes that a local-realist theory does not use interference. If there is interference, a local-realist interpretation can predict the same expectation values for a Bell test as standard quantum mechanics does, with differences for individual events like $|1,1\rangle$.

Showing that experimental results match the predicted expectation values, and closing loopholes, are not efforts against local realism, but only against local-realist theories that do not account for interference.

For pure states, such as Bell states, entanglement and inseparability are equivalent notions, in standard versions of quantum mechanics[16]. Separate probability amplitudes for ψ_A and ψ_B in Sections 3.1 and 3.2 would then cause problems for that idea.

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