Kochen-Specker and the Measurement Problem

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Abstract

The Kochen-Specker theorem rules out hidden-variable theories that are analogous to requiring a coin flip landing on heads to be heads both before and after the flip. Using a photon's linear polarization as a hidden variable, it is possible for the polarization to be in a definite state, at any angle, before a measurement. A polarizer then forces it to one of the two stable states that correspond to measurement outcomes. Such an approach might resolve the measurement problem in a general way for finite-dimensional Hilbert spaces, while a local variant of Bohmian mechanics can apply to infinite-dimensional Hilbert spaces. Entanglement can then be seen as particles that have been synchronized in the past, and decoherence as forces acting to disrupt that synchronization. The result is a noncontextual and local-realist hidden-variable interpretation.

1 Introduction

The Kochen-Specker theorem attempts to rule out noncontextual hidden-variable interpretations of quantum mechanics[1]. Section 2 shows that it bans only a subset of such theories. A noncontextual approach to the measurement problem, and a local-realist hidden-variable interpretation built on top of it, is in Section 3. Section 4 concludes.

Bell's theorem also tries to prevent certain kinds of hidden-variable theories[2]. The factorizability condition in its proof does not rule out separable states in which there is interference between the probability amplitudes of certain events[3]. The theory described in the present paper can follow such an approach, so Bell's theorem does not restrict it.

2 Kochen-Specker

2.1 Background

The basic structure of the Kochen-Specker argument is:

1) Hidden-variable theories must work a certain way.

2) Hidden-variable theories that work as in 1) produce incorrect results.

3) Any viable hidden-variable theory must have a certain property (contextuality).

Unpacking 1) leads to:

A) A classical theory uses the measurement outcome as its state.

B) A quantum theory uses probability amplitudes and the Born rule to compute probabilities of outcomes.

C) The hidden-variable theories that Kochen-Specker rules out obey A).

The Peres-Mermin square[4] provides a proof based on the above ideas. It defines an inequality that can be solved by matrices of complex numbers, as in B), but not by values that are all ± 1, as in A).

2.2 Value Definiteness

A) corresponds to the idea of value definiteness[1]. It makes sense for a hidden-variable theory to have a well-defined value for hidden variables, but it is not clear that the value has to be the same before and after a measurement. For example, comparing a quantum measurement to a coin flip[5], the coin might start out at some angle, then land on heads: it does not need to be heads both before and after the flip.

As a more relevant example, a photon might have a definite linear polarization angle before it is measured in the $(|0^\circ\rangle,\ |90^\circ\rangle)$ basis. That angle could be different than 0° or 90° before the measurement, even it must be $|0^\circ\rangle$ or blocked afterward.

If a hidden-variable theory works like that, following B) instead of A), C) no longer holds. Then, 1) fails, and the Kochen-Specker argument does not apply. For the Peres-Mermin square, that would mean a noncontextual hidden-variable theory could use matrices of complex numbers, rather than just ± 1. It could thus reproduce the standard predictions of quantum mechanics.

It seems that Bohmian mechanics may already provide an example of such a theory, when measuring position, along the lines of Section 3.3. It is a hidden-variable interpretation that computes probabilities by applying the Born rule to complex-valued probability amplitudes[6].

Section 3.2 provides more details on how the Kochen-Specker proof can fail in similar ways for finite-dimensional Hilbert spaces, following the coin flip example.

3 Measurement Problem

3.1 Background

The present paper separates the measurement problem into two cases: finite-dimensional and infinite-dimensional Hilbert spaces. The first corresponds to quantized measurements, like qubits, and the latter models continuous measurement outcomes, like position.

[7] mentions a discussion between Einstein and Schrödinger where Einstein set up a version of the EPR argument[8] as a choice between position and momentum having definite values before a measurement, or those values being created when doing a measurement.

Section 3.2 shows that, in the finite-dimensional case, there is a third option. Following Section 2.2, it approaches value definiteness with a definite value that is not necessarily an eigenstate, but which becomes one on a measurement.

The infinite-dimensional case, discussed in Section 3.3, mostly follows Bohmian mechanics. But, as shown in Section 3.4, it is possible to preserve locality.

3.2 Quantized Measurements

A quantized measurement can be interpreted as forcing a particle into one of a finite set of stable states. Those stable states correspond to the eigenstates of that measurement. The hidden variables before that measurement are well-defined, but they may differ from the eigenstates, as noted in Section 2.2.

That is in contrast to the idea that hidden-variable theories correspond to an ensemble of particles that are all in eigenstates, following some probability distribution[9].

Such measurements might be deterministic, in the same way that coin flips are: given full knowledge of the initial state, it is possible in theory to predict the outcome. There may be an appearance of randomness to an observer, due to a lack of knowledge.

In the case of quantized measurements, multiple possible initial states can result in a single final state. A unitary transformation must be invertible, and the map of initial states to final states is not invertible. That explains why measurements appear not to be unitary, when the evolution before the measurement is.

For example, a photon that passes through a vertical polarizer may have had a linear polarization before a measurement, a hidden variable, such as 20°, 30°, or another value. But, like a coin that has landed on heads, it is impossible to know the initial state just based on the measurement outcome, without having recorded some information about how it got there.

3.3 Continuous Measurements

Bohmian mechanics may be a starting point to explain continuous measurements, on infinite-dimensional Hilbert spaces[10]. In the example of the double-slit experiment, the particle's position is well-defined at all times. It takes a single path, while its associated wave may go through both gaps[6].

There it does not appear that a measurement forces a particle to a finite set of stable states. Instead, it reveals the existing value of a hidden variable. The position is well-defined at all times, and an observer's uncertainty is due to not knowing the initial state.

3.4 Local Realism

Building on [10], it is possible to use some of these ideas to create a local-realist interpretation, with one outcome per measurement, and no superdeterminism or retrocausality.

A superposition comes from a collection of well-defined hidden variables that, upon a measurement in the current basis, has probabilities of outcomes that match applying the Born rule to that state. For example, a photon with the state $\frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)$ for a measurement in the $(|0^\circ\rangle,\ |90^\circ\rangle)$ basis might correspond to a linear polarization of 45°, as a hidden variable. Superpositions are then not fundamentally different from eigenstates, where $|0\rangle$ would have a linear polarization of 0°.

Entanglement can be seen as hidden variables that have been synchronized. [11] shows that a separable state can model a Φ^+ Bell pair of photons by having the same polarization angle as a hidden variable for both photons. That is compatible with local realism, because each particle is independent of the other, and moving them apart does not require communication in order for future measurements to match the expected probabilities.

Decoherence occurs when forces act on a particle so that the values of its hidden variables change. If the hidden variables get disrupted enough, it may no longer be possible to predict measurement outcomes. The hidden variables that correspond to the initial state may have changed in ways that are difficult to predict, or by large amounts. For a Φ^+ Bell pair of polarized photons, that means the angle between the linear polarizations may change from 0°. That can reduce the probability of agreement from 1, if the forces do not disrupt both photons in the same way.

The wave-particle duality can follow the explanation from Bohmian mechanics, as in Section 3.3. Unlike Bohmian mechanics, there is no need in the present theory to use nonlocality to explain entanglement.

Thus, a number of core aspects of quantum mechanics can be explained in a straightforward way.

4 Conclusion

The Kochen-Specker theorem does not rule out noncontextual hidden-variable theories that use probability amplitudes. Section 3 shows how to build such a theory, with local realism, one outcome per measurement, and no superdeterminism or retrocausality.

References

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