A Quantum State Is an Incomplete Description

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September 12, 2026

Abstract

In a particular quantum state, it is possible for certain events either to have interference, or not to. The probabilities are different in each case, depending on whether the Born rule is applied before adding probabilities or after adding probability amplitudes. The state thus does not provide enough information to compute the correct probabilities: it is missing at least one bit, to indicate whether interference is present for those outcomes. A quantum state is therefore not a complete description of a system.

1 Introduction

A quantum state does not provide a complete description of a system: there is a state in which interference may or may not be present, and the probabilities of certain events are different in each case.

Section 2 provides an overview of the argument. The case with interference is examined in Section 3, and the one without interference in Section 4. Section 5 concludes.

2 Overview

Following [1], given constants A and B, and the separable state

$$\begin{aligned} \psi_A &= \cos(A)|0\rangle + i\sin(A)|1\rangle \\ \psi_B &= \cos(B)|0\rangle - i\sin(B)|1\rangle \\ \psi_{AB} &= \psi_A \otimes \psi_B, \end{aligned} \tag{1}$$

the state vector alone does not provide enough information to know whether there is interference for certain events in ψ_{AB}.

For those outcomes, it is not clear whether to add probability amplitudes and then apply the Born rule, or to apply the Born rule and then add probabilities.

The total probability, $P(|00\rangle \text{ or } |01\rangle \text{ or } |10\rangle \text{ or } |11\rangle)$, is 1 in each case. But, the probabilities $P(|00\rangle \text{ or } |11\rangle)$ and $P(|01\rangle \text{ or } |10\rangle)$ are different, depending on whether there is interference. Events that do not interfere with each other, like $P(|00\rangle \text{ or } |01\rangle)$, work out in both cases.

The state vector thus does not determine which set of probabilities to compute.

3 With Interference

This section considers ψ_{AB} when there is interference for $P(|00\rangle \text{ or } |11\rangle)$ and $P(|01\rangle \text{ or } |10\rangle)$.

$P(|00\rangle \text{ or } |11\rangle)$ is

$$\begin{gathered} |\cos(A)\cos(B) - i\sin(A) i\sin(B)|^2 \\ |\cos(A)\cos(B) + \sin(A)\sin(B)|^2 \\ |\cos(A-B)|^2 \\ \cos^2\big(A-B\big). \end{gathered} \tag{2}$$

$P(|01\rangle \text{ or } |10\rangle)$ is

$$\begin{gathered} |-i\cos(A)\sin(B) + i\sin(A)\cos(B)|^2 \\ |i(\sin(A)\cos(B) - \cos(A)\sin(B))|^2 \\ |i\sin(A-B)|^2 \\ \sin^2\big(A-B\big). \end{gathered} \tag{3}$$

The total probability, $P(|00\rangle \text{ or } |01\rangle \text{ or } |10\rangle \text{ or } |11\rangle)$, is 1:

$$\begin{gathered} P(|00\rangle \text{ or } |11\rangle) + P(|01\rangle \text{ or } |10\rangle) \\ \cos^2\big(A-B\big) + \sin^2\big(A-B\big) \\ 1. \end{gathered} \tag{4}$$

4 Without Interference

In contrast to Section 3, this section calculates probabilities for ψ_{AB} when there is no interference for either $P(|00\rangle \text{ or } |11\rangle)$ or $P(|01\rangle \text{ or } |10\rangle)$.

$P(|00\rangle \text{ or } |11\rangle)$ is

$$\begin{gathered} P(|00\rangle) + P(|11\rangle) \\ |\cos(A)\cos(B)|^2 + |-i\sin(A) i\sin(B)|^2 \\ |\cos(A)\cos(B)|^2 + |\sin(A)\sin(B)|^2 \\ \cos^2\big(A\big)\cos^2\big(B\big) + \sin^2\big(A\big)\sin^2\big(B\big). \end{gathered} \tag{5}$$

$P(|01\rangle \text{ or } |10\rangle)$ is

$$\begin{gathered} P(|01\rangle) + P(|10\rangle) \\ |-i\cos(A)\sin(B)|^2 + |i\sin(A)\cos(B)|^2 \\ \cos^2\big(A\big)\sin^2\big(B\big) + \sin^2\big(A\big)\cos^2\big(B\big). \end{gathered} \tag{6}$$

With

$$c = 2\cos(A)\cos(B)\sin(A)\sin(B), \tag{7}$$

$P(|00\rangle \text{ or } |11\rangle)$ without interference is c less than the equivalent probability from Section 3, and $P(|01\rangle \text{ or } |10\rangle)$ is c greater than the matching probability from Section 3.

The total probability, $P(|00\rangle \text{ or } |01\rangle \text{ or } |10\rangle \text{ or } |11\rangle)$, is again 1:

$$\begin{gathered} P(|00\rangle) + P(|01\rangle) + P(|10\rangle) + P(|11\rangle) \\ \cos^2(A)\cos^2(B) + \cos^2(A)\sin^2(B) + \sin^2(A)\cos^2(B) + \sin^2(A)\sin^2(B) \\ \cos^2(A)\left(\cos^2(B) + \sin^2(B)\right) + \sin^2(A)\left(\cos^2(B) + \sin^2(B)\right) \\ \cos^2(A) + \sin^2(A) \\ 1. \end{gathered} \tag{8}$$

5 Conclusion

In agreement with the EPR argument[2], a quantum state is an incomplete description of a system. The state is missing at least one bit, even in interpretations like many-worlds[3], to indicate whether there is interference for certain events. In a hidden-variable theory, it might be missing even more information.

References

[1] David Wyde. Separable Bell Pairs via Interference. `https://davidwyde.com/thoughts/bell-separable/`. Accessed: 2026-09-12.

[2] Albert Einstein, Boris Podolsky, and Nathan Rosen. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review*, 47(10):777–780, May 1935.

[3] Wayne Myrvold. Philosophical Issues in Quantum Theory. In Edward N. Zalta and Uri Nodelman, editors, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, Fall 2022 edition, 2022.