Quantum Interference and Bell's Theorem

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Abstract

A local realist theory can make the correct predictions for a Bell test, under a small set of assumptions: that a Bell test involves interference, a photon can have a linear polarization and certain probability amplitudes before it reaches a polarizer, and two photons in a Bell pair initially travel in opposite directions. This paper attempts to justify those claims, and to explore how and when to account for quantum interference.

1 Introduction

An earlier paper[1] argued that a local realist theory can reproduce the results of Bell tests, under the following assumptions:

1. In a Bell test based on linear polarization of photons, it is necessary to account for interference, by applying the Born rule only after doing addition and multiplication on probability amplitudes.
2. The probability amplitude of a photon at a linear polarizer, with relative angle x, is $\cos x|0\rangle + i \sin x|1\rangle$, where $|0\rangle$ is passing through and $|1\rangle$ is being blocked.
3. When two photons are in a $|\Phi^+\rangle$ Bell state, their angles for 2) have opposite signs.
4. In the local realist theory in question, a photon's polarization before a measurement is a hidden variable.

   Section 2 uses the aforementioned ideas to calculate the correct probabilities for a Bell test. Section 3 attempts to justify each of the assumptions. Section 4 includes a general discussion of quantum interference, including when to do arithmetic on probabilities versus on probability amplitudes.

2 Motivation

Given linear polarizers at angles A and B, and a Bell pair of photons each at angle y, the probability that both photons have the same outcome, $|00\rangle$ or

$|11\rangle$, is $\cos^2(A-B)$. The probability that the outcomes disagree, $|01\rangle$ or $|10\rangle$, is $\sin^2(A-B)$[2]. As described in [1], the assumptions in Section 1 lead to the expected probabilities.

The probability amplitude for agreeing is

$$\begin{aligned}
&\cos(A-y)\cos(-(B-y)) + i\sin(A-y)i\sin(-(B-y))\\
&\cos(A-y)\cos(B-y) + \sin(A-y)\sin(B-y)\\
&\cos(A-y-(B-y))\\
&\cos(A-B).
\end{aligned} \tag{1}$$

The probability amplitude for disagreeing is

$$\begin{aligned}
&i\sin(A-y)\cos(-(B-y)) + \cos(A-y)i\sin(-(B-y))\\
&i\sin(A-y)\cos(B-y) - i\cos(A-y)\sin(B-y)\\
&i\sin(A-y-(B-y))\\
&i\sin(A-B).
\end{aligned} \tag{2}$$

Applying the Born rule to each of those leads to the correct results. In contrast with Bell's theorem, the measurement at one polarizer does not depend on the setting or outcome of the other polarizer[3], due to trigonometric identities.

3 Justifying the Assumptions

3.1 Interference

The proof of Bell's theorem may have assumed a lack of interference for a Bell test: it adds expectation values, which are based on probabilities[3].

[1] shows that failing to account for interference, by multiplying and adding probabilities instead of probability amplitudes, leads to a missing term of

$$\pm 2\cos(A-y)\cos(B-y)\sin(A-y)\sin(B-y),$$

when computing the results for outcomes that agree:

$$\begin{aligned}
&\cos^2(A-y)\cos^2(B-y) + \sin^2(A-y)\sin^2(B-y)\\
\neq &\cos^2(A-B),
\end{aligned} \tag{3}$$

and that disagree:

$$\begin{aligned}
&\sin^2(A-y)\cos^2(B-y) + \cos^2(A-y)\sin^2(B-y)\\
\neq &\sin^2(A-B).
\end{aligned} \tag{4}$$

Section 4 discusses how to decide whether interference is present and when to apply the Born rule.

3.2 Probability Amplitudes

$\cos(x) + i\sin(x) = e^{ix}$ can represent plane waves that correspond to an electromagnetic field, including polarized waves[4]. The Bell test under consideration uses linear polarization of photons, and photons are individual components of light waves[5].

3.3 Opposite-Sign Angles

The e^{ix} form of a wave would have an opposite sign on x when two particles obey the same motion, but in opposite directions[6]. When a Bell pair for photon polarization is created, the two output photons have the same momentum except for opposite signs[7].

After combining those observations with the earlier notes on plane waves, it makes sense for each pair of photons in the Bell test to have one with a state of e^{ix} and the other with e^{-ix}.

The sign of the angle B is flipped, in addition to the photon that reaches that polarizer. A possible explanation involves a coordinate system with its origin where the Bell pair separated, with an axis on the line of motion of the photons, and two polarizers that each form a plane perpendicular to that line and parallel to each other.

Then, like two people facing each other while each leaning to their right, the horizontal coordinate for the angle of one polarizer corresponds to a flip over the vertical axis at the other: following the axis of motion of the photons from 90° at A will hit B at 270°. That reflection flips the sign on one polarizer's angle.

3.4 Polarization Before Measurement

The local realist theory in [1] treats photon polarization before measurement as a hidden variable.

It seems that some people and interpretations believe:

1. Light can have a linear polarization.
2. Light is made out of photons.
3. When measured, photons can act as if they have a linear polarization.
4. Photons do not have a polarization before they are measured.

Given that Bell's theorem attempts to prove that a local realist theory cannot reproduce the standard predictions of quantum mechanics[3], it seems fair to use hidden variables to construct a theory that does produce the correct results, even if not everyone believes that photons have a polarization before measurement.

4 Quantum Interference

4.1 Basic Principles

[8] says that the basic principles of computing probabilities in quantum mechanics are:

1. To turn a probability amplitude into a probability, compute the square of its absolute value.
2. When an event can occur in several indistinguishable ways, add their probability amplitudes, then apply 1) to get the probability.
3. When it is possible to determine which way an event happened, add probabilities to get a new probability.

It is not entirely clear why or when to apply the above rules: the same source also gives an alternative heuristic, to use 2) when interference is present, and 3) when there is no interference.

4.2 Handling All Cases

4.2.1 A Possible Approach

One attempt at generalizing the approaches with and without interference is:

1. For subsystems of particles where wave effects change the outcome in a predictable way, add probability amplitudes for *or*, and multiply for *and* (following the Kolmogorov axioms)[9].
2. For each event in the separate subsystems, compute the square of its absolute value to get a probability for that individual event.
3. Apply the Kolmogorov axioms to combine events from different subsystems, on the probabilities, to get final probabilities.

The idea is that waves are always present; if we can predict how they will affect the outcome, we incorporate that into our calculations, and if not, we ignore the waves entirely.

5 Conclusion

Sections 2 and 3 support the assumptions in [1], in an argument against Bell's theorem. The basic idea is to account for wave phenomena in a photon polarization Bell test. Section 4 shows, because the waves in each relevant Bell pair are synchronized in a way that can be modelled, it is important to account for interference.

References

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