Arguments Against Four No-Go Theorems

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Abstract

No-go theorems attempt to restrict how interpretations of quantum mechanics can work. Four standard examples are Bell's theorem, the Kochen-Specker theorem, Frauchiger-Renner, and Pusey-Barrett-Rudolph. The current paper provides arguments against each of those. It shows that all four fail to restrict a particular local-realist theory.

1 Introduction

Four standard quantum no-go arguments do not restrict the local-realist interpretation in [1]: Bell's theorem[2], the Kochen-Specker theorem[3], the Frauchiger-Renner argument[4], and the Pusey-Barrett-Rudolph theorem[5].

Sections 2–5 cover each of those results. Section 6 concludes.

2 Bell's Theorem

The original proof of Bell's theorem[2] assumes that a local-realist theory does not use interference[6]. That proof does not rule out local-realist theories that model Bell tests with interference.

[6] shows that it is possible to reproduce the predictions of a Bell test, with photons or a Stern-Gerlach setup on spin-$\frac{1}{2}$ particles, using a separable state: each particle in a Bell pair has a separate state vector which does not depend on the other's measurement setting or outcome.

3 Kochen-Specker

The Kochen-Specker theorem[3] requires contextuality for hidden-variable theories in which particles are in an eigenstate both before and after a measurement, as in [7]. It does not apply to the interpretation in [1].

As in [6], a photon's linear polarization can be a hidden variable. The polarization y and a detector's angle A can each be at any angle before a measurement, and the probability of passing through is $\cos^2(A - y)$. A measurement forces the photon into one of two stable states: passing through at the detector's angle, or being blocked[1].

Then, it makes sense to allow hidden-variable theories to use probability amplitudes before a measurement, rather than just eigenstates. That invalidates a standard proof of the Kochen-Specker theorem[1].

4 Frauchiger-Renner

The Frauchiger-Renner argument[4] (FR) is a version of the Wigner's Friend thought experiment. It attempts to force interpretations of quantum mechanics to give up at least one of three typical features.

The setup involves multiple measurements, with different people predicting what each other will observe. There are a number of different layers, but the simplified gambler example just involves the below steps.

In the $(|\text{heads}\rangle, |\text{tails}\rangle) \otimes (|\downarrow\rangle, |\uparrow\rangle)$ basis, the essence of FR is:

1) The initial state is $\frac{1}{\sqrt{3}}\Big(|00\rangle + |10\rangle + |11\rangle\Big)$.

2) When measuring in $|0-\rangle$, $P(|1-\rangle) = 0$: FR's (4).

3) When measuring in $|-0\rangle$, $P(|-0\rangle) = 0$: FR's (6).

4) When measuring in $|--\rangle$, $P(|--\rangle) = \frac{1}{12}$, which is nonzero.

5) Contradiction: 2), 3), and 4) are incompatible.

Given that the point of the argument is to model the setup using quantum mechanics, it is not clear how 5) is justified. Maybe 2)–4) seem counterintuitive when applied to everyday objects like people and coins, but they are what quantum mechanics predicts for the initial state in 1).

In particular, 2) does not mean that the first qubit must be $|\text{heads}\rangle$ when measured in the $(|\text{heads}\rangle, |\text{tails}\rangle)$ basis. And, 3) does not mean that the second qubit must be $|\uparrow\rangle$ when measured in the $(|\downarrow\rangle, |\uparrow\rangle)$ basis. It is possible for an inner product to be 0, for orthogonal vectors of length greater than 1, when not all of the individual products are 0. So, the intended contradiction does not work.

[8] previously showed that FR fails when modeling the experimental setup with three qubits, instead of two, using the initial coin flip's result as an extra piece of information.

5 Pusey-Barrett-Rudolph

5.1 Overview

While [9] proves that is possible for multiple physical states to map to the same quantum state, the Pusey-Barrett-Rudolph argument[5] (PBR) attempts to show that quantum mechanics prevents a single physical state from mapping to multiple quantum states.

A possible error in PBR's first proof is that it assumes a measuring device queries which state a particle was prepared in, rather than the current values of the hidden variables associated with that particle. If position is a hidden variable, a detector might measure the current position, rather than whether the initial state was $|0\rangle$, $|+\rangle$, or something else[1].

To sketch a counterexample, consider an experiment in which the initial state vector corresponds to placing a particle at a particular position within a box. The particle may move from its original position. A measurement in a given basis involves placing a detector at the same position where a particle with that state would be placed.

The measurement probabilities work as they do in standard quantum mechanics: the Born rule determines the probability of detecting other positions to which a particle might have moved, measuring in the same basis guarantees a detection, and measuring in an orthogonal basis will never have a detection. A joint measurement could involve a single device coordinating two detectors in different boxes.

Then, if it is possible for a particle placed at $|0\rangle$ to ever reach a point that can also be reached by a particle placed at $|+\rangle$, that is a counterexample to PBR.

5.2 Number Line

A different counterexample might involve a particle moving between consecutive integer coordinates along a line, with the total number of positions divisible by 4. Some measurement bases might be:

- $|0\rangle$: even numbers
- $|1\rangle$: odd numbers
- $|+\rangle$: 0 or 1 (mod 4)
- $|-\rangle$: 2 or 3 (mod 4)

Preparing a state corresponds to placing a particle at any number within that basis. A particle may then move between valid numbers in that basis. There is a 50% chance of detecting for $|0\rangle$ when prepared in $|+\rangle$, a 100% chance for $|+\rangle$, and no chance of detecting for $|-\rangle$.

Then, a measurement in a particular basis queries the current position, a hidden variable. It does not require that the particle could have been prepared in exactly one state. Measuring a particle at position 4 will result in a detection for the $|0\rangle$ or $|+\rangle$ bases, while avoiding any problems that the first PBR proof suggests will happen.

6 Conclusion

Four no-go theorems fail to constrain the local-realist interpretation in [1].

Bell's theorem and Kochen-Specker apply to a narrower range of hidden-variable theories than intended. Bell's theorem assumes that a hidden-variable theory does not use interference to describe a Bell test. A standard proof of Kochen-Specker does not apply to hidden-variable theories that use probability amplitudes.

The Frauchiger-Renner argument claims a contradiction from predictions that agree with standard quantum mechanics. A possible mistake is in the assumption that an individual event cannot occur because an inner product that includes it is 0, which is not guaranteed for vectors longer than 1.

The first proof of the Pusey-Barrett-Rudolph theorem says that a measurement outcome depends on the state in which a system was prepared, rather than that system's current state. In a theory with position as a hidden variable, that is not a safe assumption.

Thus, local-realist theories may be possible.

References

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