Quantum State Vectors

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Abstract

This paper examines four experiments in which quantum state vectors behave in surprising ways. Three approaches to modelling the Frauchiger-Renner extended Wigner's friend setup each can end up with different probabilities for the same outcomes. An attack on a quantum key exchange appears to involve the zeroing of one qubit affecting the state of another qubit, which does not make physical sense. In quantum teleportation, it is difficult to define a state where two qubits agree in the x and z bases but can have arbitrary probabilities. A simple circuit features a gate that only has an effect in certain interpretations of quantum mechanics.

1 Introduction

The present paper explores whether the standard approach to quantum state vectors sometimes fails.

Section 2 shows how adding an extra qubit can lead to outcomes that previously had probability 0. That seems to happen in [1], when modelling the extended Wigner's friend setup with 2-qubit versus 3-qubit circuits.

A circuit related to quantum key distribution seems to involve qubits affecting each other in the state vector, without interacting, in counterintuitive ways. Section 3 discusses that example.

Quantum teleportation intends to transfer the state of one qubit to another qubit. It appears that either the two qubits will disagree in the x basis, or that it is difficult for a state vector to describe two qubits that are guaranteed to agree in the x and z bases but do not have a probability of .5 for $|0\rangle$ and for $|1\rangle$. Section 4 looks at that case.

Section 5 describes a quantum circuit with a CNOT gate that only has an effect in interpretations where qubits have a state beyond probabilities or probability amplitudes.

2 Frauchiger-Renner

2.1 Basic Setup

The setup described in [1] starts with a qubit in the initial state $\sqrt{\frac{1}{3}}|H\rangle + \sqrt{\frac{2}{3}}|T\rangle$ to represent heads (H) and tails (T) of a biased coin flip. A measurement occurs on that coin, which is then used to set the spin of a second qubit to $|\downarrow\rangle$ on heads or $|\rightarrow\rangle$ on tails.

The paper then computes various probabilities based on measurements in the $|ok\rangle$ basis, $\sqrt{\frac{1}{2}}\,|\downarrow\rangle - \sqrt{\frac{1}{2}}\,|\uparrow\rangle$, and the $|\overline{ok}\rangle$ basis, $\sqrt{\frac{1}{2}}\,|H\rangle - \sqrt{\frac{1}{2}}\,|T\rangle$.

The key steps in the proof are:

1. Step (4): an initial tails cannot coincide with $|ok\rangle$.
2. Step (6): a spin-down measurement cannot coincide with $|\overline{ok}\rangle$.
3. There is a $\frac{1}{12}$ probability that both $|ok\rangle$ and $|\overline{ok}\rangle$ occur.

It seems that the initial coin flip has to be a weak measurement that preserves the coin superposition. Otherwise, if the coin is in the eigenstate of $|H\rangle$, then spin will be $|\downarrow\rangle$, $|\overline{ok}\rangle$ has a .5 probability, and (6) fails.

2.2 Different Approaches

Different ways of modelling the quantum state of the setup can sometimes cause the above proof steps to fail.

One attempt is a 2-qubit circuit (coin superposition, spin superposition), where the initial state is $\frac{|00\rangle+|10\rangle+|11\rangle}{\sqrt{3}}$. Here all 3 conclusions hold.

An alternative way to model the setup is with a 3-qubit circuit: coin superposition, spin superposition, and initial coin measurement. The initial state is $\frac{|000\rangle+|101\rangle+|111\rangle}{\sqrt{3}}$.

For the 3-qubit version, (6) seems to fail. Unlike in the original version, the start state is not orthogonal to either $|\overline{ok}\rangle \otimes |\downarrow\rangle \otimes |H\rangle$ or $|\overline{ok}\rangle \otimes |\downarrow\rangle \otimes |T\rangle$, and each has a $\frac{1}{6}$ probability of occurring.

It seems strange that having an extra qubit to track the initial coin flip measurement would lead to it suddenly becoming possible to measure a result for the other two qubits that previously had a probability of 0. The math with the state vectors works out that way, but it does not really make sense.

It is also possible that the 2-qubit and 3-qubit circuits are both incorrect. A third attempt is to use linear polarization of photons for the qubits. Then, an angle of $\arccos(\frac{1}{\sqrt{3}})$, around 54.7°, produces the original coin probabilities. The $|\overline{ok}\rangle$ basis corresponds to the $|-\rangle$, $|+\rangle$ basis, and thus has an angle of $-45°$. $\cos^2$ of the difference between those angles then results in a probability of around .029 of measuring $|\overline{ok}\rangle$.

Here a single experimental setup has multiple ways of modelling it, which might result in different measurement probabilities for the same outcomes. In the case of the 2-qubit and 3-qubit versions, adding an extra qubit to track the initial coin flip causes step (6) of [1]'s proof to fail, in a way that seems odd. It is possible that both of those models are inaccurate.

3 Quantum Key Distribution

[2] describes an attack on a quantum key exchange between Alice and Bob, who will eventually measure each of their qubits in a small number of different bases.

An eavesdropper, Eve, repeatedly intercepts a qubit X. For each choice of basis, Eve applies a gate to change X into that basis, does CNOT(X, Y) to a target qubit Y, measures Y, records a classical bit for the measurement outcome, zeroes Y, then changes X to the original basis.

Each change of basis means that $|0\rangle$ and $|1\rangle$ correspond to a measurement in that basis. Each CNOT means X and Y will agree in the z basis. Then, whichever basis Bob changes into, Eve will have a classical bit that corresponds to Bob's measurement result for that qubit, in that basis.

An alternative version of the attack has a different Y_i for each possible measurement basis. Then, Eve does a CNOT to each Y_i, passes X along to Bob, then measures each of the Y_i's, in order to reduce the delay between when Alice sends a given X and when Bob receives it.

The state vector for the latter approach suggests that the attack will fail: there can be a nonzero probability that Eve will get a result that disagrees with Bob's.

It is then worth asking what would happen if Eve removes each Y_i from the circuit, instead of measuring and zeroing it, before changing X back from each target basis to the original basis.

In both the multi-qubit attack and the version in which the Y_i's are removed at each step, it seems that every Y_i should be independent from all of the other Y_i's. Zeroing or measuring one of them should not affect the others, regardless of what kind of entanglement is present, as they are separate qubits.

It seems that unrelated qubits somehow affect each other in the state vector, in ways that do not make physical sense.

4 Quantum Teleportation

4.1 Basic Setup

This section analyzes a quantum teleportation setup in which Alice has a photon C in some state, A and B are in a $|\Phi^+\rangle$ Bell state, Alice receives A and Bob receives B, and the goal is to transfer C's initial state to Bob's B.

In the standard teleportation process[3], Alice does a Bell measurement on A and C, then sends the resulting classical bits to Bob, and Bob does a CNOT and a controlled Z gate with those bits as the source and B as the target. The final state vector indicates that B ends in the same state that C initially was in.

4.2 Analysis

The CNOT in the Bell measurement, then the CNOT on B, will make B agree with C in the z basis. That would explain why the state vector is the same for both.

The form of a two-qubit state that is guaranteed to agree in both the x and z bases is $a\left|00\right\rangle + a\left|11\right\rangle$, such that $|a|^2 = .5$. The state for agreeing in the z basis is $c\left|00\right\rangle + d\left|11\right\rangle$. Solving a linear system for $\mathrm{H} \otimes \mathrm{H}$, which is its own inverse, multiplied by that state, results in

$$(.5a + .5b)\left|00\right\rangle + (.5a - .5b)\left|01\right\rangle + (.5a - .5b)\left|10\right\rangle + (.5a + .5b)\left|11\right\rangle.$$

Guaranteed agreement in the x basis then requires $\left|01\right\rangle$ and $\left|10\right\rangle$ to have probability amplitudes of 0 there. That forces $a = b$, which results in the aforementioned state.

If B and the original C become $a\left|00\right\rangle + b\left|11\right\rangle$ after the CNOT on B, based on the initial C being $a\left|0\right\rangle + b\left|1\right\rangle$, and a and b are not equal, it does not make sense that B would agree with the original C in the x basis.

On the other hand, the teleportation circuit seems to check whether A and C agree in the x and z bases, then uses that information to flip B if it disagrees with C. It then seems possible that the state vector cannot describe a setup where two qubits are guaranteed to agree in both the x and z bases but have a probability other than .5 for each measurement in the z basis.

It is thus not clear that the teleportation circuit works as intended, despite what the state vector indicates.

5 Gates With No Effect

To create a quantum circuit that is subject to different interpretations, it is possible to start in $\left|00\right\rangle$, run an H gate on both qubits, then run a CNOT. All of the probability amplitudes are .5 before and after the CNOT: it swaps $\left|10\right\rangle$ and $\left|11\right\rangle$, but they have the same value.

In an interpretation of quantum mechanics where qubits have a physical state before a measurement, the CNOT has changed something: if the outcome would have been $\left|10\right\rangle$, it is now $\left|11\right\rangle$, and vice versa.

In an interpretation where probability amplitudes or probabilities are the fundamental reality, the CNOT does not appear to have changed anything about the world.

In this example, whether a particular gate has any effect is open to debate.

6 Conclusion

In each of the case studies above, quantum state vectors appear to behave in surprising ways.

References

[1] Daniela Frauchiger and Renato Renner. Quantum theory cannot consistently describe the use of itself. *Nature Communications*, 9, 2018. Article number: 3711.

[2] David Wyde. Quantum Key Distribution and the Uncertainty Principle. `https://davidwyde.com/thoughts/qkd-uncertainty/`. Accessed: 2025-10-07.

[3] Charles H. Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William K. Wootters. Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels. *Physical Review Letters*, 70(13):1895–1899, 1993.